A New Finite-Element Solution for Parameter Extraction of Multilayer and Multiconductor Interconnects

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***Abstract*— In this letter, the geometry-independent measured equation of invariance is adopted with the finite-element method for mesh truncation and is successfully used in parameter extraction of interconnects. The major advantage is overcoming the tedious closed-form Green's function deduction and disagreeable Sommerfeld integral calculation for multilayer and multiconductor structures. Moreover, the high-order finite-element is employed to increase the accuracy and save computer resources. Furthermore, an optimizing numerical scheme, which is found to be very efficient, is developed to solve finite-element equations.**

***Index Terms*—Finite element, interconnect, measured equation of invariance.**

I. Introduction

THE recently developed measured equation of invariance (MEI) is a new boundary condition which is very different from other previously used absorbing boundary conditions (ABC's), such as perfectly matched layer (PML) [1], [2] and asymptotic boundary condition (ABC) [3], etc. The implementation of MEI in finite-element method (FEM) has appeared in electromagnetic scattering calculation [4], [5], however, to our knowledge, no application has been given yet for parameter extraction of interconnects. In this letter, the FEM in conjunction with the geometry-independent MEI (GIMEI) is presented for interconnects analysis. Moreover, the high-order finite-element (HOFE), which can make the mesh truncated more closely to the body and therefore save the computer resources, is employed in the calculation. Furthermore, in numerical scheme the substructure-front (S-F) technique is utilized for the first time to optimize the solution of FEM for interconnects.

We would like to mention here that this letter is inspired by some published references [4]–[7], however, it differs from those references in four important aspects.

1) The GIMEI is used with FEM instead of traditional MEI.
2) The HOFE is adopted in the calculation.
3) The parameter extraction of multilayer and multiconductor interconnect is specified.
4) The S-F technique is introduced in numerical scheme.

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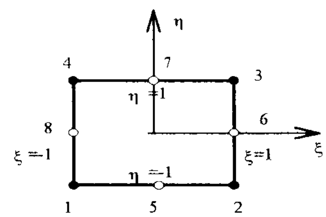

Fig. 1. High-order finite-element (HOFE) model.

II. High-Order Finite Element in MEI Method

Since the frequency range of interest for high-speed VLSI is often below 20 GHz, we adopt the quasi-TEM assumption. Thus, the governing equation for interconnect is

$$\nabla^2 U = 0. \tag{1}$$

For the two-dimensional (2-D) field, we choose the HOFE known as the eight-node square isoparametric element (see Fig. 1). The correspondent node shape function is

$$\begin{aligned} N_i^e(\xi,\eta) &= \tfrac{1}{4}(1+\xi_i\xi)(1+\eta_i\eta) \\ &\quad\cdot(\xi_i\xi+\eta_i\eta-1), \qquad i=1,2,3,4 \\ N_i^e(\xi,\eta) &= \tfrac{1}{2}(1-\xi^2)(1+\eta_i\eta), \qquad i=5,7 \\ N_i^e(\xi,\eta) &= \tfrac{1}{2}(1-\eta^2)(1+\xi_i\xi), \qquad i=6,8. \end{aligned} \tag{2}$$

According to the MEI method, the boundary condition for an open boundary node is

$$\sum_{i=1}^{M} \beta_i U_i = 1. \tag{3}$$

In this work, we choose $M = 4$. By utilizing HOFE as described above, the adjacent three nodes were chosen as shown in Fig. 2. It is worth mentioning that for the "vertex" node in HOFE seen in Fig. 2(c), another "vertex" node Q is chosen instead of "middle" node P as the number 4 node, because this letter is based on the principle that one of the three adjacent nodes must be on the connecting surface between MEI-boundary element and interior element, when considering that there should exist a coupling relation between MEI elements and interior elements. In general, when using HOFE in FEM/MEI, the following rules should be mentioned.

- For nonboundary element, only the standard FE equation is used.

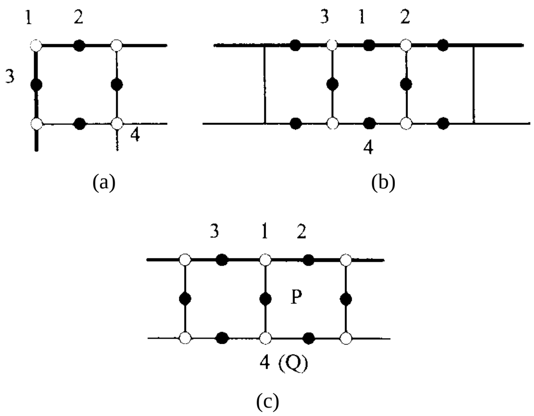

Fig. 2. Boundary node number 1 and adjacent three nodes for HOFE in MEI (a) for "corner" node, (b) for "middle" node, and (c) for "vertex" node.

- For MEI-boundary element, if the node is an interior node adjacent to the boundary node, the FE equation still can be used; if the node is a boundary node, the MEI condition (3) is used.

III. FEM/GIMEI METHOD

It is known that MEI theory is based on the postulates which are geometry specific and location dependent, thus, for complex structure of interconnects, the closed-form Green's functions are generally derived in spectral domain and then transformed to the space domain by inverse Fourier transformation, which are infinite integrals. In addition to the undesirable deduction of Green's function, the calculation of MEI coefficients is very time consuming because many Sommerfeld integrals will be encountered. In order to overcome these difficulties, we introduce the GIMEI concept proposed by Hong *et al.* [7]. The description of the main idea is that a measuring loop is introduced to isolate the MEI elements from the region containing conductors, and then the MEI coefficients are determined by the metrons on the measuring loop rather than on the conductors, which indicates that the MEI conditions are independent on the conductors and media enclosed by the measuring loop. This implies that we abandon the postulate of specified geometry in the MEI concept. In the meantime, we assume that the outer space outside the loop is free space, thus the free-space Green's function can still be applied and the time of coefficients calculation sequentially decreases greatly. In this work, we find that five-layer meshes between the measuring loop and the left and the right conductors, and two-layer meshes outside the loop, are sufficient to guarantee accuracy.

In the FEM/GIMEI method, we first divide the whole region into two sub-regions. One is the FE region including all elements except MEI-boundary elements; the other is MEI region. In the calculation, if the element is of the FE region, it is just treated in the usual way, so a triangular matrix is used to store the FE matrix coefficients for constructing the global matrix considering the symmetry of each finite-element matrix. If on the other hand the element is of the MEI region, since MEI conditions have no symmetric properties, a full matrix should be used to store the MEI coefficients.

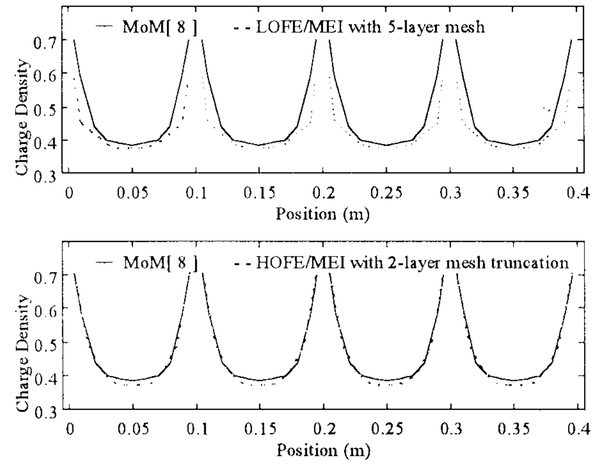

Fig. 3. The charge distribution on an infinite PEC 0.1-m-square cylinder raised to 10-V potential as predicted by (HOFE/MEI), (LOFE/MEI), and MoM (unit of charge density: $\times 10^{-9}$).

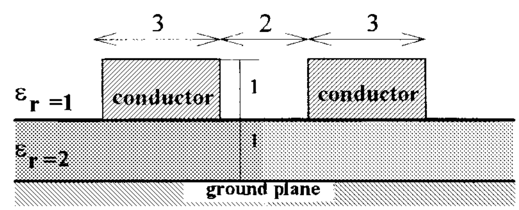

Fig. 4. A numerical example of interconnects.

In order to optimize the numerical solution of the FE equations, the S-F technique incorporating the FEM/GIMEI, which includes four main steps, is specially developed. First, the whole region is divided into the MEI region and the FE region, including the interesting region (i.e., the surface enclosing the conductors). Second, the elements in the FE region are processed using the Front Method, where only those variables of interesting region and the terminal surface between the MEI and FE regions remain in the "front." Third, the present "front" is transformed into a full matrix to prepare for the incorporation of MEI conditions. Fourth, MEI elements are dealt with in the full matrix and are solved together. In fact, the number of unknowns to be solved in the S-F technique can be decreased substantially, therefore the solution process is fast.

IV. NUMERICAL RESULTS

Example 1: It can be seen from Fig. 3 that the result of HOFE is more accurate than that of low-order finite-element (LOFE), and the meshes of HOFE are terminated more closely to the conductor.

Example 2: The parameter extraction of interconnect for a multiconductor transmission line (see Fig. 4) has been done by FEM/GIMEI, and the results of capacitance are given in Table I. We find that our results are very close to those of other authors. Additionally, it should be noted that by using the S-F technique, the memory usage of the S-F method is only one-twentieth of Gauss method and its CPU time is just about one-twelfth of Gauss methods.

TABLE I
COMPARISON OF CAPACITANCE BETWEEN OUR METHOD AND OTHER METHODS (UNIT: pF/m)

	our method	Hong's[7]	Cao's[9]	Weeks'[10]
C11	93.69	93.80	91.65	92.24
C12	-8.27	-8.32	-8.22	-8.50
C21	-8.27	-8.32	-8.22	-8.50
C22	93.69	93.80	91.65	92.24

V. CONCLUSIONS

The proposed approach overcomes the difficulties associated with disagreeable Green's function deduction for multilayer and multiconductor structures and the Sommerfeld-type integral calculation. Moreover, the application of HOFE and S-F technique are sufficient to guarantee the accuracy and speed up the computation of interconnects. Therefore, this letter presents an efficient finite-element solution for parameter extraction of interconnects.

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